12 Hasse Diagram. Lattice.

83. Given is the relation $R = \{(a, b) \mid a \leq b\}$ on the set $S = \{1, 2, 3, 4\}$.

(i) Show that R is a partial order relation.

(ii) Sketch the Hasse diagram.

(iii) Does S have the structure of a lattice with respect to the relation R? (Justify your answer!)

84. Let $S = \{1, 2, \ldots, 10\}$. A relation on the set S is defined as follows:

$$xRy \quad \Leftrightarrow \quad x + y \text{ is even and } x \leq y.$$

(i) Show that R partially orders the set S.

(ii) Draw the Hasse diagram with respect to R.

(iii) Find all R-maximal elements, if they exist.

(iv) Does S have the structure of a lattice with respect to R?

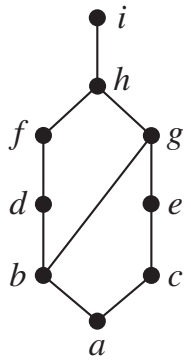
85. Given is a partial order relation $\{(A, B) \mid A \subseteq B\}$ defined on the power set $\mathcal{P}(S)$ of the set $S = \{a, b, c\}$.

(a) Sketch the Hasse diagram.

(b) Find the lower and upper bounds for the subset $\{\{a\}\}$.

(c) Find the lower and upper bounds for the subset $\{\{a, c\}\}$.

(d) Find the lower and upper bounds for the subset $\{\{a\}, \{a, c\}, \{c\}\}$.



86. The partial order relation $\preccurlyeq$ on the set $S = \{a, b, c, d, e, f, g, h, i\}$ is defined by the Hasse diagram on the left.

(a) Find all $\preccurlyeq$-lower bounds and the $\preccurlyeq$-greatest lower bound of the subset $\{d, e\}$, if they exist.

(b) Find all $\preccurlyeq$-upper bounds and the $\preccurlyeq$-least upper bound of the subset $\{f, c\}$, if they exist.

(c) Determine whether S has the structure of a lattice with respect to $\preccurlyeq$. Justify your answer in detail.

87. (2nd colloquium, January 2022) Let the relation of divisibility R be defined on the set $S = \{2, 4, 5, 6, 7, 10, 14, 18, 20, 24, 25\}$:

$$xRy \qquad \Leftrightarrow \qquad x \text{ divides } y.$$

(a) Draw the Hasse diagram with respect to R.

(b) Find all R-maximal elements, if they exist.

(c) Find all R-minimal elements, if they exist.

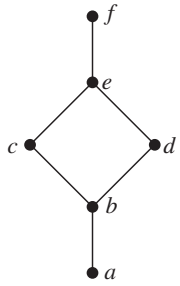
(d) Find all R-lower bounds of the subset $\{24\}$, if they exist.

(e) Find the R-greatest lower bound of the subset $\{24\}$, if it exists.

(f) Determine whether S has the structure of a lattice with respect to R.

88. Draw the Hasse diagram of the partial order $(D(60), |)$. Here, $D(60) = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$ and $|$ is the relation of divisibility. Also, find $|$-sup$(\{20, 30\})$, $|$-sup$(\{2, 6\})$, $|$-sup$(\{2, 15\})$, $|$-inf$(\{20, 30\})$, and $|$-inf$(\{6, 5\})$, if they exist.

89. The partial order R on the set S is given by the following Hasse diagram.



Does the set S have the structure of a lattice with respect to R? Justify your answer in detail.

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
The reader can find all solutions to the given problems on the same page.